TRINITY COLLEGE

MATHEMATICS METHODS UNITS 3&4

Section One: Calculator-free

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free   **35% (52 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1   **(5 marks)**

A summary of the lengths of a large sample of nails from a production line are shown below.

Length, L mm	Relative frequency
$147 < L \leq 148$	0.17
$148 < L \leq 149$	0.13
$149 < L \leq 150$	0.21
$150 < L \leq 151$	0.19
$151 < L \leq 152$	0.16
$152 < L \leq 153$	0.14

(a)   What proportion of nails are longer than 149 mm?   (1 mark)

(b)   Determine the probability that a randomly selected nail from the production line is longer than 150 mm given that it is no longer than 152 mm.   (2 marks)

(c)   State, with reasons, whether the data suggests that the nail lengths are normally distributed.   (2 marks)

Question 2   **(5 marks)**

(a)   Determine $\int \dfrac{2x+2}{x^2+2x-3}\,dx, \ x > 1$.   (2 marks)
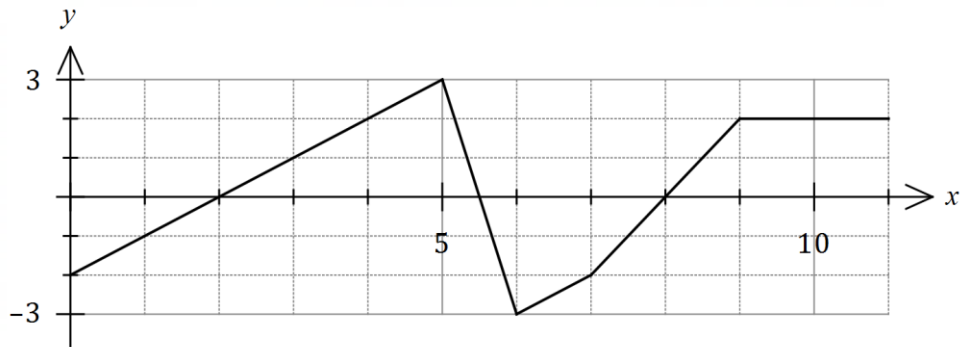
(b)   The line $y = 10 - 2x$ intersects the curve $y = \dfrac{8}{x}$ at $(1,8)$ and $(4,2)$. Determine the area
       trapped between line and the curve.   (3 marks)

Question 3 (7 marks)

The graph of $y = f(x)$ consists of line segments, as shown below.

Evaluate each of the following:

(a) $\displaystyle\int_3^5 f(x)\, dx.$ (1 mark)

(b) $\displaystyle\int_0^5 f(x)\, dx.$ (2 marks)

(c) $\displaystyle\int_2^7 2f(x)\, dx.$ (2 marks)

(d) $\displaystyle\int_5^9 (f(x) + 1)\, dx.$ (2 marks)

See next page

Question 4 (7 marks)

The curve $y = 4x + \dfrac{2}{x^2}$ has one stationary point.

(a) Obtain expressions for $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. (2 marks)

(b) Determine the coordinates of the stationary point and determine its nature. (4 marks)

(c) Explain why the curve has no point of inflection. (1 mark)

See next page

Question 5 **(7 marks)**

(a) Let $F(x) = \displaystyle\int_0^x \cos 3\theta \, d\theta$.

Express $F(x)$ as a function of x and hence evaluate $F\left(\dfrac{\pi}{4}\right)$.

(3 marks)

(b) Let $g(x) = \dfrac{e^{1-x}}{1-x}$.

(i) Show that $g'(x) = \dfrac{x \, e^{1-x}}{(1-x)^2}$. (2 marks)

(ii) Hence, or otherwise, evaluate $\displaystyle\int_{-1}^{0} \dfrac{2x \, e^{1-x}}{(1-x)^2} \, dx$. (2 marks)

See next page

Question 6   **(7 marks)**

(a) By first using log laws, or otherwise, determine $\frac{d}{dx}\left(\ln\left(e^{2x}\sqrt{x^3+1}\right)\right)$ in simplest form.

(3 marks)

(b) The function $f(x) = x^2 \ln\left(\frac{x}{2}\right)$ for $x > 0$ has one stationary point, a global minimum.

Determine the minimum value of the function.   (4 marks)

See next page

Question 7 (8 marks)

The random variable X is defined by $P(X = x) = \begin{cases} k \log_3(x + 2) & x = 1, 25, 79 \\ \\ 0 & \text{elsewhere} \end{cases}$

(a) Determine the value of the constant k. (2 marks)

(b) Calculate the expected value of X. (2 marks)

The Bernoulli random variable Y is solely dependent on X, so that $Y = 1$ when $X = 1$, and $Y = 0$ for all other values of X.

(c) Determine

  (i) $P(Y = 0)$. (1 mark)

  (ii) $E(Y)$. (1 mark)

  (iii) $\text{Var}(3Y + 1)$. (2 marks)

See next page

Question 8 **(6 marks)**

In triangle ABC, the length a of the side opposite angle A is given by $a = \sqrt{11 - 4\cos A}$ cm.

Use the increments formula to calculate the approximate change in length of a as the size of angle A decreases from $\dfrac{15\pi}{45}$ to $\dfrac{14\pi}{45}$.

End of questions

Supplementary page

Question number: _____